$$S(x, t) = \frac{1}{2} \exp(-at) \left\{ \varphi(x - vt) + \varphi(x + vt) + \right.$$

$$\left. + a \int_{x-vt}^{x+vt} \varphi(\xi) \left[\frac{1}{v} I_0 \left(\frac{a}{v} \sqrt{v^2t^2 - (\xi - x)^2} \right) + \frac{t I_1 \left(\frac{a}{v} \sqrt{v^2t^2 - (\xi - x)^2} \right)}{\sqrt{v^2t^2 - (\xi - x)^2}} \right] d\xi \right\},$$

where I_0, I_1 are Bessel functions of imaginary argument of the zeroth and first order, respectively. All the appropriate concentration profiles are here in agreement to 1.5ε accuracy, which indicates the efficiency of the computation method examined above.

NOTATION

t, time; x, coordinate; P, probability; a, characteristic frequency of turbulent pulsations; D, coefficient of turbulent diffusion; R', integer part of the number R/2; and R" = R–R'.

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SOLUTION OF HEAT-CONDUCTION PROBLEMS IN HETEROGENEOUS MEDIA BY THE INTEGRAL RELATIONS METHOD

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The integral relations method is developed to solve heat-conduction problems in a two-component complex medium and nonstationary filtration for the case of bounded and unbounded domains.

The integral relations method is used sufficiently extensively to solve heat-conduction problems because of the simplicity of the method itself and of the approximate solutions obtained with its use. A detailed survey and application of this method to solve different linear and nonlinear heat-transfer problems in homogeneous bodies can be found in [1]; this method is also applied in other branches of the mechanics of continuous media, e.g., in the theory of nonstationary filtration [2].

The method of integral relations has not been used to solve heat-conduction problems in heterogeneous continuous media; however, its application to filtration problems in binary media has been attempted (the equations of heat propagation in heterogeneous media [3] are analogous to the equations of nonstationary filtration of a homogeneous fluid in porous-cracked media [4, 5]). A completely degenerate system of heat-conduction equations in a two-component continuous medium ($\varepsilon_1 = 0$, $\varepsilon_2 = 0$), reduced to one equation, was taken as the basis in [6]. In such an approach it is required to take account of the singularity in the formulation of the initial and boundary conditions [7, 8], which is inconvenient, and also the domain of application of the method is shrunken (the condition $\varepsilon_1 \approx 0$ is not always satisfied). Moreover, the dimension of the perturbed zone turns out to be different from zero at the ini-

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tial time in [6], which is physically inexplicable. The paper [9] is devoid of the disadvantages listed above, however the integral relations there are inaccurately compiled; utilization of such relations to solve nonlinear problems results in physically impossible results, the dimension of the perturbed zone turns out to be a complex number at a certain time. Let us also note that the integral relations method has generally not been used to solve heat-conduction problems in heterogeneous media for domains of finite size.

1. The system of heat-conduction equations for a two-component continuous medium has the following form in the case of axial symmetry

$$\varepsilon_1 \frac{\partial T_1}{\partial \theta} - q(\rho,\ \theta) = \chi \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial T_1}{\partial \rho},$$

$$\frac{\partial T_2}{\partial \theta} + q(\rho,\ \theta) = \varepsilon_2 \chi \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial T_2}{\partial \rho}, \tag{1}$$

$$\theta = t/\tau,\ \rho = r/R,\ \chi = a\tau/R^2,$$

$$\varepsilon_1 = \frac{m_1\gamma_1 c_1}{m_2\gamma_2 c_2},\quad \varepsilon_2 = \frac{m_2\lambda_2}{m_1\lambda_1},\quad a = \frac{m_1\lambda_1}{m_2\gamma_2 c_2},$$

$$\tau = m_2\gamma_2 c_2/\alpha. \tag{2}$$

Let the initial and boundary conditions be

$$\theta = 0:\ T_1 = T_2 = 0,$$

$$\rho \to 0: \rho \frac{\partial}{\partial \rho}(T_1 + \varepsilon_2 T_2) = \overline{Q}(\theta),$$

$$\rho \to 0:\ T_1 = T_2. \tag{3}$$

Just as in solving the ordinary Fourier equation by the integral relations method, we introduce moving perturbation zones l_1 and l_2 in the medium consisting of particles of the first component (we call this the first medium), and in the medium consisting of particles of the second component (the second medium), and consider the exterior of the perturbation zones unperturbed. The conditions on the boundary of these zones are

$$\rho = l_i(\theta):\ T_i = 0,\ \frac{\partial T_i}{\partial \rho} = 0,\ i = 1,\ 2. \tag{4}$$

We seek the distribution of the desired quantities in the form

$$T_i = A_i \ln \rho/l_i + B_i + C_i \rho/l_i, \tag{5}$$

which, taking account of the boundary conditions and of (4), yields

$$T_i = Q(\theta)(1 + \ln \rho/l_i - \rho/l_i),\ Q(\theta) = \overline{Q}(\theta)/(1 + \varepsilon_2). \tag{6}$$

To determine l_i we use (1) to compile integral relations:

$$\varepsilon_1 \int_0^{l_1} \rho \frac{\partial T_1}{\partial \theta} d\rho - \int_0^{l_1} \rho q(\rho,\ \theta) d\rho = \chi\rho \frac{\partial T_1}{\partial \rho}\Big|_0^{l_1},$$

$$\int_0^{l_2} \rho \frac{\partial T_2}{\partial \theta} d\rho + \int_0^{l_2} \rho q(\rho,\theta) d\rho = \varepsilon_2\chi\rho \frac{\partial T_2}{\partial \rho}\Big|_0^{l_2}. \tag{7}$$

Taking into account that

$$q(\varphi,\ \theta) = T_2(\rho,\ \theta) - T_1(\rho,\ \theta),$$

we will have

$$\int_0^{l_1} \rho q(\rho,\ \theta) d\rho = \int_0^{l_2} \rho T_2 d\rho + \int_{l_2}^{l_1} \rho T_2 d\rho - \int_0^{l_1} \rho T_1 d\rho, \tag{8}$$

$$\int_0^{l_2} \rho q(\rho, \theta)\, d\rho = \int_0^{l_2} \rho T_2 d\rho - \int_0^{l_1} \rho T_1 d\rho + \int_{l_2}^{l_1} \rho T_1 d\rho. \tag{9}$$

One expression is used in [9] in place of (8) and (9)

$$\int_0^{l_2} \rho T_2 d\rho - \int_0^{l_1} \rho T_1 d\rho. \tag{10}$$

Utilization of (10) instead of (8) and (9) is inaccurate as will be shown below.

The physical meaning of introducing the perturbation zones is the following: it is considered that the perturbation produced on the boundary in addition to the heat transfer between the media will produce a moving perturbation zone $l_1(\theta)$ in the first medium, and $l_2(\theta)$ in the second. The appropriate medium is considered unperturbed outside these zones. Hence, to compile the integral relations each of Eqs. (1) is integrated strictly within the limits of the appropriate zones.

It should be noted that the use of (9) is inconvenient since the last term in the right side of (9) will have a sufficiently complex form when (6) is substituted. This is easily averted if instead of (1) as initial system, an equivalent system is taken:

$$\varepsilon_1 \frac{\partial T_1}{\partial \theta} + \frac{\partial T_2}{\partial \theta} = \chi \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} (T_1 + \varepsilon_2 T_2),$$

$$\varepsilon_1 \frac{\partial T_1}{\partial \theta} + T_1 - T_2 = \chi \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial T_1}{\partial \rho}. \tag{11}$$

Then the integral relations are written as

$$\varepsilon_1 \int_0^{l_1} \rho \frac{\partial T_1}{\partial \theta} d\rho + \int_0^{l_2} \rho \frac{\partial T_2}{\partial \theta} d\rho = \chi\rho \frac{\partial T_1}{\partial \rho}\Big|_0^{l_1} + \varepsilon_2 \chi\rho \frac{\partial T_2}{\partial \rho}\Big|_0^{l_2},$$

$$\varepsilon_1 \int_0^{l_1} \rho \frac{\partial T_1}{\partial \theta} d\rho + \int_0^{l_1} \rho (T_1 - T_2)\, d\rho = \chi\rho \frac{\partial T_1}{\partial \rho}\Big|_0^{l_1}. \tag{12}$$

As is customary, the integral relations are expanded by using the formulas for differentiation of a definite integral with respect to a parameter and taking into account that the second medium is considered unperturbed in the interval l_1–l_2. We obtain

$$\varepsilon_1 \frac{dz_1}{d\theta} + \frac{dz_2}{d\theta} = 12\chi (1 + \varepsilon_2) Q(\theta),$$

$$\varepsilon_1 \frac{dz_1}{d\theta} + z_1 - z_2 = 12\chi Q(\theta),$$

$$z_i = Q(\theta)\, l_i^2(\theta), \tag{13}$$

which is easily solved. We obtain for the case Q = const

$$l_1^2 = \frac{12\chi}{1+\varepsilon_1} \left\{ (1 + \varepsilon_2)\theta + \frac{1 - \varepsilon_1\varepsilon_2}{1+\varepsilon_1} \left[1 - \exp\left(-\frac{1+\varepsilon_1}{\varepsilon_1} \theta \right) \right] \right\},$$

$$l_2^2 = \frac{12\chi}{1+\varepsilon_1} \left\{ (1 + \varepsilon_2)\theta - \varepsilon_1 \frac{1 - \varepsilon_1\varepsilon_2}{1+\varepsilon_1} \left[1 - \exp\left(-\frac{1+\varepsilon_1}{\varepsilon_1} \theta \right) \right] \right\}. \tag{14}$$

The equality of the perturbation zone to zero at the initial time is used to determine the constant of integration.

If we set $\varepsilon_1\varepsilon_2 \approx 1$ in (14), then the dependences (14) go over into a formula for a homogeneous continuous medium with certain total parameters:

$$l_1^2 = l_2^2 = l^2 = \frac{12\chi(1+\varepsilon_2)}{1+\varepsilon_1}\theta. \tag{15}$$

2. Since the perturbations in the first medium are propagated more rapidly than in the second, then we have three phases of the process in a bounded domain, rather than the two as in a homogeneous medium:

a) The first phase, that l_1 and l_2 have still not reached the domain boundary (process in an infinite domain), was examined above;

b) The second phase is when l_1 has reached the domain boundary but l_2 not yet;

c) The third phase is when l_2 has reached the domain boundary.

It is easy to see that the construction of the solution for the second and third phases of the process will also reduce to integrating a system of type (13). For instance, if we have a circular plate of radius $\rho = 1$ with a heat-insulated outer boundary, then distribution T_2 in the second phase remains in form (6), but T_1 is sought in the form

$$T_1 = b_1(\theta) + Q(\theta)(1 + \ln\rho - \rho). \tag{16}$$

From the integral relations

$$\varepsilon_1 \int_0^1 \rho \frac{\partial T_1}{\partial \theta} d\rho + \int_0^{l_2} \rho \frac{\partial T_2}{\partial \theta} d\rho = \chi\rho \frac{\partial T_1}{\partial \rho}\Big|_0^1 + \varepsilon_2\chi\rho \frac{\partial T_2}{\partial \rho}\Big|_0^{l_2},$$

$$\varepsilon_1 \int_0^1 \rho \frac{\partial T_1}{\partial \theta} d\rho + \int_0^1 \rho (T_1 - T_2)\, d\rho = \chi\rho \frac{\partial T_1}{\partial \rho}\Big|_0^1 \tag{17}$$

we determine $b_1(\theta)$ and $l_2(\theta)$. The constants of integration are found from the continuity of the passage from one phase to another, as in the case of a homogeneous medium [1, 2].

For Q = const, the solution has the form

$$b_1 = -\frac{2\chi Q(1+\varepsilon_2)}{1+\varepsilon_1}(\theta - \theta_1) + \frac{2\chi Q}{1+\varepsilon_1}\left[\frac{1-l^*}{12\chi} - \frac{1-\varepsilon_1\varepsilon_2}{1+\varepsilon_1}\right]\left\{1 - \exp\left[-\frac{1+\varepsilon_1}{\varepsilon_1}(\theta - \theta_1)\right]\right\}, \tag{18}$$

$$l_2^2 = l^* + \frac{12\chi(1+\varepsilon_2)}{1+\varepsilon_1}(\theta - \theta_1) + \frac{12\varepsilon_1\chi}{1+\varepsilon_1}\left[\frac{1-l^*}{12\chi} - \frac{1-\varepsilon_1\varepsilon_2}{1+\varepsilon_1}\right]\left\{1 - \exp\left[-\frac{1+\varepsilon_1}{\varepsilon_1}(\theta - \theta_1)\right]\right\}. \tag{19}$$

The duration of the first phase of the process θ_1 is determined from the first equation of (14) for $l_1 = 1$; l^* is determined from the second equation of (14) for $\theta = \theta_1$. For the third phase of the process both quantities are sought in the form

$$T_i = b_i(\theta) + Q(\theta)(1 + \ln\rho - \rho). \tag{20}$$

From the integral relations

$$\varepsilon_1 \int_0^1 \rho \frac{\partial T_1}{\partial \theta} d\rho + \int_0^1 \rho \frac{\partial T_2}{\partial \theta} d\rho = \chi\rho \frac{\partial}{\partial \rho}(T_1 + \varepsilon_2 T_2)\Big|_0^1,$$

$$\varepsilon_1 \int_0^1 \rho \frac{\partial T_1}{\partial \theta} d\rho + \int_0^1 \rho (T_1 - T_2)\, d\rho = \chi\rho \frac{\partial T_1}{\partial \rho}\Big|_0^1 \tag{21}$$

we determine $b_1(\theta)$ and $b_2(\theta)$, and for the case of Q = const the solution has the form

$$b_1 = -\frac{2\chi Q(1+\varepsilon_2)}{1+\varepsilon_1}(\theta - \theta_2) - \frac{1}{1+\varepsilon_1}\left[\frac{2\chi Q(1-\varepsilon_1\varepsilon_2)}{1+\varepsilon_1} + b^*\right]\left\{1 - \exp\left[-\frac{1+\varepsilon_1}{\varepsilon_1}(\theta - \theta_2)\right]\right\} + b^*, \tag{22}$$

$$b_2 = -\frac{2\chi Q(1+\varepsilon_2)}{1+\varepsilon_1}(\theta - \theta_2) + \frac{\varepsilon_1}{1+\varepsilon_1}\left[\frac{2\chi Q(1-\varepsilon_1\varepsilon_2)}{1+\varepsilon_1} + b^*\right]\left\{1 - \exp\left[-\frac{1+\varepsilon_1}{\varepsilon_1}(\theta - \theta_2)\right]\right\}. \tag{23}$$

The constants of integration for (21) are determined from the condition of continuity of passage of the second phase of the process into the third. The duration of the second phase θ_2 is determined from (19) for $l_2 = 1$; b^* is determined from (18) for $\theta = \theta_2$. The first terms in the right sides of the dependences (22) and (23) agree with the expression for a homogeneous continuous medium with certain total parameters.

3. The integral relations method can be used effectively to solve nonlinear systems of the type (1). We show its application in the example of a known problem form the theory of filtration, the problem of ideal gas filtration in a porous-cracked stratum. There is no exact solution of this problem at this time.

Plane-parallel filtration of an ideal gas in a porous-cracked stratum is described by the following system of equations:

$$\varepsilon_1 \frac{\partial P_1}{\partial \theta} + \frac{\partial P_2}{\partial \theta} = \varkappa \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} (P_1^2 + \varepsilon_2 P_2^2),$$

$$\varepsilon_1 \frac{\partial P_1}{\partial \theta} + P_1^2 - P_2^2 = \varkappa \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial P_1^2}{\partial \rho},$$

$$\varkappa = a^* \tau_0 / R^2, \; a^* = (k_1 P_0)/(2 m_2 \mu),$$

$$\varepsilon_1 = m_1/m_2, \; \varepsilon_2 = k_2/k_1, \; \theta = t/\tau_0. \tag{24}$$

We can arrive at the system (24) in problems of heat conduction in a two-component complex medium under the assumption that the properties of the components depend as follows on the temperature:

$$\lambda_1 = \lambda_{1_0} (1 + \beta T_1) \quad (\beta = \text{const}),$$

$$\lambda_2 = \lambda_{2_0} (1 + \beta T_2),$$

$$\alpha = \alpha_0 [(1 + \beta T_1) + (1 + \beta T_2)].$$

In the case of starting up a borehole with a constant mass debit in an initially unperturbed stratum, the initial and boundary conditions for (24) are written thus:

$$\theta = 0: P_1^2 = P_2^2 = 1,$$

$$\rho = u_0: \rho \frac{\partial}{\partial \rho} (P_1^2 + \varepsilon_2 P_2^2) = M_0 = \text{const},$$

$$\rho = u_0: P_1^2 = P_2^2;$$

$$u_0 = \delta_1(\theta)/l_1(\theta) = \delta_2(\theta)/l_2(\theta), \; u_0 \ll 1. \tag{25}$$

Let us introduce two fictitious moving borehole radii $\delta_1(\theta)$, $\delta_2(\theta)$ in the first and second media, respectively (such a method is used in the case of filtration in an ordinary porous medium, say, [10]).

Giving the distribution of the desired quantities in the form

$$P_i^2 = A_i \ln \rho/l_i + B_i + C_i \rho/l_i \tag{26}$$

and taking account of the boundary conditions and the conditions

$$\rho = l_i(\theta): \begin{array}{l} P_i^2 = 1, \\ \dfrac{\partial P_i^2}{\partial \rho} = 0, \end{array} \tag{27}$$

we obtain

$$P_i^2 = 1 + M(1 + \ln \rho/l_i - \rho/l_i); \; M = M_0/(1 + \varepsilon_2)(1 - u_0). \tag{28}$$

Using (24) and (28), we compile the integral relations exactly as for the linear equations, but we take into account that the quantity P_2^2 between l_1 and l_2 equals 1 in this case. Expanding the integral relations, we obtain

$$\varepsilon_1 \frac{d}{d\theta} [l_1^2 (0.5 - I)] + \frac{d}{d\theta} [l_2^2 (0,5 - I)] = \varkappa (1 + \varepsilon_2) M, \tag{29}$$

$$\varepsilon_1 \frac{d}{d\theta}[l_1^2(0.5-I)] + \frac{M}{12}(l_1^2 - l_2^2) = \varkappa M,$$

$$I = \int_{u_0}^{1} u[1 + M(1 + \ln u - u)]^{1/2} du; \quad u = \rho/l_i. \tag{30}$$

The integral I is tabulated [10]. Solving (29), we obtain

$$l_1^2 = \frac{\varkappa}{1+\varepsilon_1}\left\{\frac{(1+\varepsilon_2)M}{0.5-I}\theta + 12\frac{1-\varepsilon_1\varepsilon_2}{1+\varepsilon_1}\left[1 - \exp\left[-\frac{(1+\varepsilon_1)M}{12\varepsilon_1(0.5-I)}\theta\right]\right]\right\},$$

$$l_2^2 = \frac{\varkappa}{1+\varepsilon_1}\left\{\frac{(1+\varepsilon_2)M}{0.5-I}\theta - 12\varepsilon_1\frac{1-\varepsilon_1\varepsilon_2}{1+\varepsilon_1}\left[1 - \exp\left[-\frac{(1+\varepsilon_1)M}{12\varepsilon_1(0.5-I)}\theta\right]\right]\right\}. \tag{31}$$

As for the linear equations, (31) goes over into a dependence for a homogeneous medium for $\varepsilon_2 = 1$ (in this case it is always possible to set $\varepsilon_1 = 1$ in filtration theory)

$$l_1^2 = l_2^2 = l^2 = \frac{\varkappa M(1+\varepsilon_2)}{(1+\varepsilon_1)(0.5-I)}\theta. \tag{32}$$

Let us note yet another circumstance. With great accuracy the integral I can be represented in the form [10]

$$I = 0.5 + \omega_1 M \text{ — pumping},$$
$$I = 0.5 - \omega_2 M \text{ — selection},$$
$$\omega_1 = \text{const}, \ \omega_2 = \text{const}, \ \omega_1 \neq \omega_2. \tag{33}$$

It is then clearly seen from (31) that the processes of gas pumping in the stratum (corresponds to the heating process in heat conduction theory) and selection occur in a different way that does not hold for the linearized equations. For an ordinary porous medium this fact follows from the exact solution of the Boussinesq equation, as Polubarinova-Kochina [11] and Barenblatt [12] indicated.

Let us note that if the integral relations for the problem considered here are compiled in conformity with [9], then the solution will have the form

$$l_1^2 = \frac{\varkappa M}{1+\varepsilon_1}\left\{\frac{1+\varepsilon_2}{0.5-I}\theta - \frac{12(1-\varepsilon_1\varepsilon_2)}{(1+\varepsilon_1)(6-M)}\left[1 - \exp\left[\frac{(1+\varepsilon_1)(6-M)}{12\varepsilon_1(0.5-I)}\theta\right]\right]\right\},$$

$$l_2^2 = \frac{\varkappa M}{1+\varepsilon_1}\left\{\frac{1+\varepsilon_2}{0.5-I}\theta + \frac{12\varepsilon_1(1-\varepsilon_1\varepsilon_2)}{(1+\varepsilon_1)(6-M)}\left[1 - \exp\left[\frac{(1+\varepsilon_1)(6-M)}{12\varepsilon_1(0.5-I)}\theta\right]\right]\right\}. \tag{34}$$

It is seen from (34) that at a certain time the quantity l_2^2 becomes negative; from physical considerations this is false. Moreover, l_1 and l_2 in (34) depend on the parameter M that characterizes the boundary conditions, which should also not hold. Dependences (31) are devoid of these inconsistencies, therefore, the integral relations should be compiled just as has been mentioned above (see (7)-(9), (12)).

4. For nonlinear heat-conduction equations in heterogeneous media in a bounded domain, the scheme for constructing solutions for the second and third phases of the process remains exactly the same as for linear equations. However, in this case the construction of the solution reduces to integrating a nonlinear system of a differential equations, which can reduce to a Riccati equation in the example considered in Sec. 3, and later to a linear second order equations whose solution is expressed in terms of special functions. The detailed calculations are expounded in [13], where it is shown that an asymptotic representation of these functions can be used.

If the problem examined in Sec. 3 is supplemented by the condition

$$\rho = 1: \ \frac{\partial P_i^2}{\partial \rho} = 0, \tag{35}$$

then the solution of this problem for the third phase of the process (which is often of greatest interest) will have the following form [13]:

$$P_i^2 = b_i^2(\theta) + M(1 + \ln\rho - \rho), \tag{36}$$

$$b_1 = \frac{1+\varepsilon_1 b^*}{1+\varepsilon_1} - \frac{2\varkappa M(1+\varepsilon_2)}{1+\varepsilon_1}(\theta-\theta_2) - \frac{(1-\varepsilon_1\varepsilon_2)\varkappa M}{(1+\varepsilon_1)[1+\varepsilon_1 b^* - 2\varkappa M(1+\varepsilon_2)(\theta-\theta_2)]}, \tag{37}$$

$$b_2 = \frac{1+\varepsilon_1 b^*}{1+\varepsilon_1} - \frac{2\varkappa M(1+\varepsilon_2)}{1+\varepsilon_1}(\theta-\theta_2) + \frac{\varepsilon_1(1-\varepsilon_1\varepsilon_2)\varkappa M}{(1+\varepsilon_1)[1+\varepsilon_1 b^* - 2\varkappa M(1+\varepsilon_2)(\theta-\theta_2)]}. \tag{38}$$

NOTATION

t, time, sec; τ, lag time, sec; m_i, part of the volume occupied by the appropriate component (porosity of the i-th medium in the filtration problem); r, polar coordinate, m; c_i, specific heat, J/kg·deg; γ_i, density, kg/m³; R, distance scale, m; λ_i, thermal conductivity W/m·deg; a, thermal diffusivity coefficient, m²/sec; $\overline{Q}(\theta)$, parameter characterizing the boundary conditions; $l_i(\theta)$, relative size of the moving perturbation zone; T_i, temperature measured from the initial temperature and referred to a certain temperature scale; θ_1, θ_2, durations of the first and second phases of the process, respectively; l^*, square of the relative size of the perturbation zone in the second medium at the instant of the beginning of the second phase of the process; b*, relative temperature in the first medium on the heat-insulated outer boundary at the instant of beginning the third phase of the process (gas pressure in the first medium on the outer impermeable boundary at the time of beginning the third phase of the process in the filtration problem); k_i, permeability, m², u, gas viscosity, Pa·sec; a^*, coefficient of piezoconduction, m²/sec; p_i, gas pressure referred to the pressure in the initially unperturbed stratum; p_0, pressure in the initially unperturbed stratum, Pa; M_0, parameter characterizing the boundary conditions. Subscripts: 1, first medium; 2, second medium.

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